The Dirac Equation and the Two Photon Model of the Electron revised

D.T. Froedge

Formerly Auburn University
dtfroedge@glasgow-ky.com

V041621
@ http://www.arxdtf.org

Abstract

In a previous paper, "The Electron as a Composition of Two Vacuum Polarization Confined Photons", included here as Appendix I [1]. The model shows structure, and the physical description required for the existence of an electron being a composition of two photons bound by the self-induced vacuum polarization gradient of the index of refraction. This physical model of the electron makes the mathematics more compatible with Lorentzian mechanics, and forms the basis for the interaction between the Feynman's sum-over-paths and electric charge.

This paper and its revision will focus on the mathematical structure of the electron including the spin aspects of its bound structure.

Background and Basis

Dirac

The Dirac expression factored by the use of Geometrical Algebra factors the wavefunction into a product of functions. This split is somewhat artificial in that the solutions are factors of a single wavefunction of two particles. The factored solutions have some unrealistic properties including negative energy and imaginary components that should be real. The problems primarily results from the roots of negative quantities. The equation is created out the Klein Gordon Equation with an additional requirement to satisfy the symmetry of spacetime, and there is no physical basis for the existence of the equation as a defining equation for the electron.

P-Dirac

The approach here is to first define wavefunctions of the individual photons in the electron having 4-derivitive that are proper Lorentz 4-vectors momentum. The magnitude of this sum can then be shown to have the properties of the electron including mass and spin.

The magnitude of the sum of two 4-vectors is the root of the square of the sum, thus if two proper Lorentz 4-vectors are summed the magnitude is the products are a Lorentz scalar invariant. For a single photon this is zero, but if there is a change in the index if refraction the magnitude is not zero, and there is rest mass component [2]. With the proper values of energy, and phases, the electron as a Lorentz scalar invariant rest energy as the sum of two photons can be derived.

This legitimately creates an equation for two bound photons from the well-known relativistic dynamics, having a proper rest mass and spin.

Photon Wavefunctions

The photon wavefunctions of the two photons are proposed to be a space-time function such that:

$$\psi_1 = e^{i\left(\frac{\hat{k}\bullet x}{\lambda_1} - \omega_1 t\right)} = e^{i\left(\frac{m_1 c_1}{\hbar}\hat{k}\bullet x - \frac{m_1 c_0}{\hbar} ct\right)} \qquad \psi_2 = e^{i\left(-\frac{\hat{k}\bullet x}{\lambda_2} - \omega_2 t\right)} = e^{i\left(-\frac{m_2 c_1}{\hbar}\hat{k}\bullet x - \frac{m_2 c}{\hbar} c_0 t\right)} \quad (1)$$

Where m is defined as $m = \hbar\omega / c^2$ and $\hat{k}$ is the unit vector in the direction of travel. The photons are set, as shown, to be phase conjugates. The subscripts on c_0 refers to the velocity of light referenced to the observers frame and c_1, is the velocity of light in another frame or medium with an index of refraction different from the observer. [3],[4],[5],[6]. A change in the index of refraction c_1 / c_0 does not change the frequency of the time component of the wavefunction.

The momentum 4-vector for the photon is the four derivative of the wavefunction:

$$p^\mu = \hbar\gamma^\mu \frac{\partial}{\psi_1 \partial x^\mu} e^{i\left(\frac{mc_1}{\hbar}\hat{k}\bullet x - \frac{mc_0}{\hbar} c_0 t\right)} \quad (2)$$

The four space vector p^μ is the standard four-space relativistic momentum In the Weil representation normally used to define the 4-space momentum of a photon. With, $c_1 = c_0$, the relativistic scalar magnitude is zero:

$$E_I = \hbar c \left| \gamma^{\mu} \frac{\partial}{\psi \partial x^{\mu}} e^{i\left(\frac{mc_1}{\hbar}\hat{k}\bullet x - \frac{m_1 c_0}{\hbar} ct\right)} \right| = 0 \tag{3}$$

The magnitude is the square root of the product of the vector times its conjugate.

If $c_1 = c_0$ the value of E_I vanishes thus there is no invariant rest energy, but if the location of the photon moves into a higher index of refraction such as piece of glass where $c_1 \neq c_0$ then the product or square of the four vectors is not zero, but a Lorentz invariant scalar [2], and the photon has not only relativist kinetic energy, but also a scalar invariant rest energy. This invariant rest energy creates rest mass equivalent in the rest frame of the glass. When it leaves the glass, the energy is again, pure relativistic energy, thus the glass has by slowing the velocity temporarily changed the energy from relativistic to invariant.

Though the wavefunctions may be strange to some, Eq.(2), and Eq.(3), are well understood Quantum methods for evaluation the momentum and energy of a photon.

The value of the invariant rest energy of a photon can be shown from the scalier magnitude of the resulting 4-momentum. Thus if the photon is in a location with a reduced value of c:

$$\Delta c_1 = c_0 - c_1 \quad \rightarrow \quad c_1 = c_0 - \Delta c_1 \tag{4}$$

, and

$$\vec{p}_1 = \hbar \left[\gamma^{\mu} \frac{\partial}{\psi \partial x^{\mu}} e^{i\left(\frac{m_1 c_0}{\hbar}(x - c_0 t) - \frac{m_1 \Delta c_1}{\hbar} x\right)} \right] \tag{5}$$

$$= \left| \hbar \left(\gamma^{\mu} \left(\frac{m_1 c_0}{\hbar} - \frac{m_1 c_0}{\hbar} \right) - \gamma^{k} \frac{m_1 \Delta c_1}{\hbar} \right) \right| = \hbar c \left(\frac{m_1 \Delta c_1}{\hbar} \right) \tag{6}$$

The product of two Lorentz four-vectors is a Lorentz scalar, [2], thus the magnitude of the invariant mass energy in Eq.(6), is added to the glass in its rest frame is:

$$E_I = m_1 c_0 \Delta c_1 \tag{7}$$

This is an important concept since it is clear that if a photon is slowed down or trapped in a volume of space, there is an invariant scalar rest mass present.

Three Parts of the Energy of a Photon

From the above discussion, the components of the total energy are; the invariant or rest energy E_I, the kinetic energy E_K, and the total energy, E_T, specifically this is:

$$E_I = c_0\hbar\left|p^\mu\right| = c_0\hbar\left|\gamma^\mu \frac{\partial\psi}{\psi\partial x}\right| \qquad E_K = c_0\hbar\left|\gamma^k \frac{\partial\psi}{\psi\partial x}\right| \qquad E_T = c_0\hbar\left|\gamma^0 \frac{\partial\psi}{\psi\partial x}\right| \tag{8}$$

The difference of the kinetic and total energy is the Lorentz invariant scalar rest mass.

Summing Two Photon Momentum

The sum of two Lorentz 4-vectors is also a Lorentz 4-vector, thus the magnitude of the momentum of two photons is a scalar invariant.

Presume $p_1 + p_2$ is the sum of the momentum for two photons, then the invariant energy for the sum is:

$$E_I = c_0\hbar\left|\left(p + p\right)\right| \tag{9}$$

, or from Eq.(8):

$$E = c_0\hbar\left|\gamma^\mu \frac{\partial\psi_1}{\psi_1\partial x} + \gamma^\mu \frac{\partial\psi_2}{\psi_1\partial x}\right| \tag{10}$$

Consider the two identical phase conjugate photons of Eq.(1), which are moving in opposite directions:

$$p_1 = \hbar\left[\gamma^\mu \frac{\partial}{\partial x^\mu} e^{i\left(\frac{m_1 c_0}{\hbar}\left(k \bullet x - c_0 t\right)\right)}\right] \qquad p_2 = \hbar\left[\gamma^\mu \frac{\partial}{\partial x^\mu} e^{i\left(\frac{m_2 c_0}{\hbar}\left(-k \bullet x - c_0 t\right)\right)}\right] \tag{11}$$

The invariant energy of the sum is:

$$E_{\mathrm{I}} = \mathrm{c}\hbar\left|\left(p_1 + p_2\right)\right| \tag{12}$$

, or

$$E_{\mathrm{I}} = c\hbar\sqrt{\left(p_1 + p_2\right)\left(p_1 + p_2\right)^*} = \mathrm{c}\hbar\sqrt{\left(p_1 p_1^* + p_2 p_2^* + p_2 p_1^* + p_1 p_2^*\right)} \tag{13}$$

Both are Lorentz 4-vectors with a zero scalar value thus.

$$p_1 p_1^* = 0 \qquad\qquad p_2 p_2^* = 0 \tag{14}$$

The cross terms however are not necessarily zero, but are Lorentz scalar invariants [2]. If $p_1^{\mathrm{k}} \bullet p_2^{\mathrm{k}*}$ is positive or the photons are in opposite directions, there is a scalar invariant rest mass. ($\gamma^{\mathrm{k}} \bullet \gamma^{\mathrm{k}} = -1$)

$$\begin{aligned} p_1 p_2^* &= \left(p_1^{\mathrm{k}} \bullet p_2^{\mathrm{k}*} + p_1^0 p_2^{0*}\right) = 2\mathrm{m}_1\mathrm{m}_2\mathrm{c}_0^2 \\ p_2 p_1^* &= \left(p_2^{\mathrm{k}} \bullet p_1^{\mathrm{k}*} + p_2^0 p_1^{0*}\right) = 2\mathrm{m}_1\mathrm{m}_2\mathrm{c}_0^2 \end{aligned} \tag{15}$$

In this case the cross terms are alike and:

$$p_1 p_2^* = p_2 p_1^* \tag{16}$$

If the value of m_1 and m_2 is equal to the two internal photons in the electron [1] $\mathrm{m}_{\mathrm{e}} = \mathrm{m}_1 + \mathrm{m}_2$ then:

$$p_1 p_2^* + p_2 p_1^* = \mathrm{m}_{\mathrm{e}}^2\mathrm{c}_0^2 \tag{17}$$

The energy is then:

$$E_{\mathrm{I}} = \mathrm{c}\hbar\sqrt{p_1 p_1^* + \; p_2 p_2^* + p_1 p_2^* + p_2 p_1^*} \;=\; \sqrt{4\mathrm{m}_1\mathrm{m}_2}\,\mathrm{c}_0^2 \;=\; \mathrm{m}_{\mathrm{e}}\mathrm{c}_0^2 \tag{18}$$

Thus:

$$E_{\mathrm{I}} = \mathrm{c}\hbar\sqrt{\;2 p_1 p_2^*} \;=\; \mathrm{m}_{\mathrm{e}}\mathrm{c}_0^2 \tag{19}$$

The value of the invariant scalar rest mass of the sum of the two opposing conjugate phase photons is that of the electron. If the two photons are confined such as in bound orbital motion as in the electron (Noted appendix I), this is a Lorentz invariant scalar measurable rest mass.

Connection to the Dirac Equation

Eq.(19), and Eq.(11), could now be restated as:

$$2\gamma^{\mu}\frac{\partial\psi_1}{\psi_1\partial x}\times\gamma^{\mu}\frac{\partial\psi_2{}^{*}}{\psi_2{}^{*}\partial x}=\frac{m_e^2c_0^2}{\hbar^2} \tag{20}$$

This is the magnitude of the sum of the momentum of two conjugate phase photons, having energy equivalent to the electron, and is identical to the factored Dirac expression.

Dirac Equation

$$\left(\gamma^{\mu}\frac{\partial}{\partial x^{\mu}}+\frac{m_e c}{\hbar}\right)\psi_1\times\left(\gamma^{\mu}\frac{\partial}{\partial x^{\mu}}-\frac{m_e c}{\hbar}\right)\psi_2=0$$

$$\text{or}\quad \gamma^{\mu}\frac{\partial\psi_1}{\psi_1\partial x^{\mu}}\times\gamma^{\mu}\frac{\partial\psi_2}{\psi_2\partial x^{\mu}}=\frac{m_e^2c_0^2}{\hbar} \tag{21}$$

The Dirac equation Eq.(21), represents two mass particles one having a positive inertial mass, and one of which has a negative inertial mass. The negative mass thought by Dirac to be the positron, but the positron also has a positive inertial mass.[7],[8].

P-Dirac

Eq.(20), designated as P-Dirac is an alternate to the Dirac equation and is the basis of our earlier assertion that the electron is the binding of two rotating, opposite going phase conjugate photons. (See Included Paper). The Dirac equation, as well as the Schrodinger equation are cast as a starting point for the electron, and electron interaction, and do not have a physical basis. The P-Dirac equation is the magnitude of the sum of the 4-momentum of **two proper Lorentz relativistic photon momentums that have an invariant rest mass** [2].

Photon-Photon Interaction and Spin

As has been noted the photons in the electron are held in orbit by the gradient in the speed of light induced by the probable action paths of the other photon [1]. These mutually induced increases in the index of refraction bind the photons together and function as a potential between the photons, and creates the spin components of the eigenfunctions.

The momentum of the photon, $p = \mathrm{mc}$ is a function of c, thus the angular momentum of the photon is a function of the index of refraction. (See Appendix I)

This change in c can be included in the wave equation of the photon, (Eq.(6)), and thus is included in the momentum. The change in c is a space vector and $\mathrm{k} \bullet \mathrm{x}$ can accommodate this change

$$\gamma^{\mathrm{k}} \frac{\partial}{\partial \mathrm{x}^{\mathrm{k}}} \mathrm{k} \bullet \mathrm{x} \rightarrow p + \Delta p \tag{22}$$

By noting that $\mathrm{c} = \mathrm{c}_0 - (\mathrm{c}_0 - \mathrm{c}) = \mathrm{c}_0 - \Delta \mathrm{c}$, the change can be included in the momentum of the photons as noted in Eq.(6). If this is included in both p_1 and p_2. The properties of the internal properties of the electron can be developed as:

$$p_1 p_2{}^* = -\frac{\mathrm{m}_1 \mathrm{c}_0}{\hbar}\left(\left(\gamma^{\mathrm{k}} - \gamma^0\right) - \gamma^{\mathrm{k}}_{1\perp}\frac{\Delta \mathrm{c}}{\mathrm{c}}\right)\frac{\mathrm{m}_2 \mathrm{c}}{\hbar}\left(\left(+\gamma^{\mathrm{k}} + \gamma^0\right) - \gamma^{\mathrm{k}}_{2\perp}\frac{\Delta \mathrm{c}}{\mathrm{c}}\right) \tag{23}$$

$$p_2 p_1{}^* \rightarrow -\frac{\mathrm{m}_2 \mathrm{c}_0}{\hbar}\left(\left(-\gamma^{\mathrm{k}} - \gamma^0\right) + \gamma^{\mathrm{k}}_{2\perp}\frac{\Delta \mathrm{c}}{\mathrm{c}}\right)\frac{\mathrm{m}_1 \mathrm{c}_0}{\hbar}\left(\left(-\gamma^{\mathrm{k}} + \gamma^0\right) + \gamma^{\mathrm{k}}_{1\perp}\frac{\Delta \mathrm{c}}{\mathrm{c}}\right) \tag{24}$$

For the change in the momentum of one photon by the other as they turn in a circle the change c is perpendicular to p Thus we can designate $\gamma^{\mathrm{k}}_{1\perp}$ being perpendicular to γ^{k}.

As shown earlier the change in c is proportional to the electric potential thus the relation of the binding force to an electric equivalent can be evaluated and shown equal to the Schwinger electric energy density.

Detailed of the calculations are included in Appendix I

$$p_1 p_2{}^* = \frac{1}{2}\mathrm{m}_{\mathrm{e}}^2 \mathrm{c}_0^2 \left(1 + \frac{\Delta \mathrm{c}}{\mathrm{c}_0}\boldsymbol{\sigma}\right) \tag{25}$$

$$p_2 p_1^* = \frac{1}{2}\mathrm{m}_e^2\mathrm{c}_0^2\left(1 - \frac{\Delta \mathrm{c}}{\mathrm{c}}\boldsymbol{\sigma}\right) \tag{26}$$

The sum is the same as Eq.(17), Which is the expected value of the rest mass,

$$p_2 p_1^* + p_2 p_1^* = \mathrm{m}_e^2\mathrm{c}_0^2 \tag{27}$$

The commutator of the interaction is a time independent spin vector

$$p_2 p_1^* - p_2 p_1^* = \mathrm{m}_e^2\mathrm{c}_0^2\frac{\Delta \mathrm{c}}{\mathrm{c}_0}\boldsymbol{\sigma} \tag{28}$$

The interaction terms of the magnitude of the momentum, Eq.(25), and Eq.(26), are just the two component eigenfunctions of the Dirac equation with the change in c, $\Delta\mathrm{c}$, induced by the vacuum polarization.

Noting that from the Attached Paper, regarding the two photon model, Eq.(1,17), the change in c at the electron orbit as a result of the spin and index of refraction to bind the photon is:

$$\frac{\Delta \mathrm{c}}{\mathrm{c}_0} = \frac{1}{2} \tag{29}$$

The commutator of the two eigenfunctions is then:

$$p_2 p_1^* - p_2 p_1^* = \frac{1}{2}\mathrm{m}_e^2\mathrm{c}_0^2\sigma \tag{30}$$

Spin in the Dirac and the P-Dirac Equations

Dirac was able to deduce the spin and angular momentum of the electron by applying an electromagnetic field to the equation and noting the difference in the effect on the two eigenfunctions, as a result of a potential generated by the angular momentum. This separated the effect of the angular momentum from the total momentum allowing evaluation.

In the two photon model it is easy to see the contribution of the angular momentum to each of the photon interaction terms Eq.(25), and Eq.(26). Additionally, from the two photon model, the spin can calculated directly (Attached Paper [1]), Eq.(1,8))

Discussion

The eigenfunctions of the two primary solutions of the P-Dirac equation $p_2 p_1^*$ and $p_1 p_2^*$ are directly identified with of the Dirac eigenfunctions solutions each of which have the same spin as found by Dirac. The mass energy for both the photon interaction terms is positive, whereas for the Dirac expression one is negative.

The relation between the Dirac equation Eq.(21), and the two photon Equation developed here is clear. The Dirac equation is an operation on a single wavefunction for two particles; it has two solutions one of which has negative energy, and an imaginary electric moment. It is suggested here that the problem is that the single function solution requires the presence of square roots which generate negative and imaginary values in the solutions.

The solution presented here, (P-Dirac) is of the product of two photon momenta constituted as the 4 gradient of well-defined photon wavefunctions. By reducing the velocity (or increasing the index of refraction) of the photons by a change in c necessary to provide a stable orbit generated by the Schwinger vacuum polarization, an additional spin term occurs in each of the interaction eigenfunctions. These terms are identical to those in each the Dirac solutions. The terms are opposite thus for the sum the interaction terms the spin vanishes, but the commutator is the sum of these two spin matrix indicating it is silent unless the particle is subjected to the presence of an electromagnetic field. Including the electromagnetic four-potential the photon momentum of Eq.(23), or Eq.(24), produces the same fictitious moments.

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Appendix I

Detail Calculations of the Photon Cross Terms of $p_1 p_2^*$ and $p_2 p_1^*$

$$p_1 p_2^*$$

$$p_1 p_2^* = -\frac{\mathrm{m}_1 \mathrm{c}_0}{\hbar}\frac{\mathrm{m}_2 \mathrm{c}}{\hbar}\left(\left(\gamma^{\mathrm{k}} - \gamma^0\right) - \gamma^{\mathrm{k}}_{1\perp}\frac{\Delta \mathrm{c}}{\mathrm{c}}\right)\left(\left(+\gamma^{\mathrm{k}} + \gamma^0\right) - \gamma^{\mathrm{k}}_{2\perp}\frac{\Delta \mathrm{c}}{\mathrm{c}}\right)$$

$$p_1 p_2^* = -\hbar^2 \frac{\mathrm{m}_1 \mathrm{c}_0}{\hbar}\frac{\mathrm{m}_2 \mathrm{c}}{\hbar}\begin{bmatrix} +\left(\gamma^{\mathrm{k}} - \gamma^0\right)\left(+\gamma^{\mathrm{k}} + \gamma^0\right) + \left(\gamma^{\mathrm{k}}_{1\perp}\frac{\Delta \mathrm{c}}{\mathrm{c}}\gamma^{\mathrm{k}}_{2\perp}\frac{\Delta \mathrm{c}}{\mathrm{c}}\right) \\ -\gamma^{\mathrm{k}}_{1\perp}\frac{\Delta \mathrm{c}}{\mathrm{c}}\left(+\gamma^{\mathrm{k}} + \gamma^0\right) - \left(\gamma^{\mathrm{k}} - \gamma^0\right)\gamma^{\mathrm{k}}_{2\perp}\frac{\Delta \mathrm{c}}{\mathrm{c}} \end{bmatrix} \quad (31)$$

$$p_1 p_2^* = -\frac{1}{4}\mathrm{m}_e^2 \mathrm{c}_0^2 \begin{bmatrix} -2 + \frac{\Delta \mathrm{c}}{\mathrm{c}}\frac{\Delta \mathrm{c}}{\mathrm{c}}\gamma^{\mathrm{k}}_{1\perp}\gamma^{\mathrm{k}}_{2\perp} \\ +\frac{\Delta \mathrm{c}}{\mathrm{c}}\left[-\left(+\gamma^{\mathrm{k}}_{1\perp}\gamma^{\mathrm{k}} + \gamma^{\mathrm{k}}_{1\perp}\gamma^0\right) - \left(\gamma^{\mathrm{k}}\gamma^{\mathrm{k}}_{2\perp} - \gamma^0\gamma^{\mathrm{k}}_{2\perp}\right)\right] \end{bmatrix} \quad (32)$$

Note that in the circular orbit of eh photons $+\gamma^{\mathrm{k}}_{1\perp}$ is perpendicular to γ^{k}, the photon momentum thus , $\gamma^{\mathrm{k}}_{1\perp}\gamma^{\mathrm{k}} = 0$, and the space vector and the time vector anti-commute thus, $\gamma^{\mathrm{k}}_{1\perp}\gamma^0 = -\gamma^{\mathrm{k}}_{2\perp}\gamma^0$.The product of $\gamma^{\mathrm{k}}_{2\perp}\gamma^0 = \boldsymbol{\sigma}$, is the Dirac spin vector.

The value of the square is: $\left(\frac{\Delta \mathrm{c}}{\mathrm{c}}\frac{\Delta \mathrm{c}}{\mathrm{c}}\right) = 2\frac{\Delta \mathrm{c}}{\mathrm{c}}$, thus the Expression becomes:

$$p_1 p_2^* = \frac{1}{2}\mathrm{m}_e^2 \mathrm{c}_0^2 \left(1 + \frac{\Delta \mathrm{c}}{\mathrm{c}_0}\boldsymbol{\sigma}\right) \quad (33)$$

$$p_2 p_1^*$$

With the same procedures for $p_2 p_1^*$ the results are:

$$p_2 p_1^* \to - \frac{m_2 c_0}{\hbar}\frac{m_1 c_0}{\hbar}\left(\left(-\gamma^k - \gamma^0\right) + \gamma^k_{2\perp}\frac{\Delta c}{c}\right)\left(\left(-\gamma^k + \gamma^0\right) + \gamma^k_{1\perp}\frac{\Delta c}{c}\right) \quad (34)$$

, or:

$$p_2 p_1^* \to -\frac{1}{4} m_e^2 c_0^2 \left(-\left(2 + 2\frac{\Delta c}{c} \right) + 2\frac{\Delta c}{c}\gamma^k_{2\perp}\gamma^0 \right) \quad (35)$$

, and

$$p_2 p_1^* \to -\frac{1}{4} m_e^2 c_0^2 \left(-\left(2 + 2\frac{\Delta c}{c} \right) + 2\frac{\Delta c}{c}\gamma^k_{2\perp}\gamma^0 \right) \quad (36)$$

$$p_2 p_1^* = \frac{1}{2} m_e^2 c_0^2 \left(1 - \frac{\Delta c}{c}\boldsymbol{\sigma} \right) \quad (37)$$

Attached Paper
Included Below

Attached Paper

The Electron as a Composition of Two Vacuum Polarization Confined Photons Revised II

This is the second revision of this paper and it resulting from discovering some errors when developing the two photon version of the Dirac equation (P-Dirac)

D.T. Froedge

Formerly Auburn University
dtfroedge@glasgow-ky.com

V022620
@ http://www.arxdtf.org

Abstract

In a previous paper "A Physical Electron-Positron Model"[1] an electron model was developed in a geometrical algebra (GA) construct developed by Doran et.al. [2] The model shows the mathematical structure, and the physical description required for the existence of an electron as a composition of two photons bound by the self-induced vacuum polarization gradient of the index of refraction. This paper will develop the mechanics of a vacuum polarization induced index of refraction, binding photons in orbit around a common center of momentum.

The concept of charge has heretofore not had a theoretical explanation, accept for some unknown substance associated with mass. This model offers the physical concept of charge created from QFT mechanics.

> If a plane polarized photon wavefunction is turned into a circle by a gradient in the index of refraction induced by a second photon, and this circle circumference is half the wavelength, then the wave polarization is constant along the radial axis, and the photon electric vector can be said to be radially polarized.

Introduction

The wave particle duality of particle dynamics is understood as physical aspects of particles that require both perspectives to predict the outcome of experimental tests. For the purposes of this paper we will subscribe to the wave nature of a photon as a prediction of the probability location, and the particle as a point particle with dynamics directed by a gradient in the speed of light induced by the nonlinear aspects of vacuum polarization. The physical photon is assumed to be very small, on the order of the Planck particle.

The wave nature of the electron has been well developed by Schrodinger, Dirac and many others. The Lagrangian wave nature alone however is inadequate to describe well known measurable phenomena such as charge, size and mass

By appealing to the particle nature and the nonlinear effects of photon interaction, a composition particle that has, mass, spin, and size can be developed. The electron size has been a particularly difficult issue for QFT since there is an infinite singularity associated with the electron. This model should be useful in regard to resolving some of those issues. The individual photons in the model still have singular aspects, but not the infinities associated with the electron.

In Geometric Algebra (GA) the Dirac Matrices becomes the spacetime unit coordinate vectors, which indirectly changes the normal view of QM by defining some of the aspects QM as actually features of Lorentz covariant spacetime. Parity, time reversal, and mass, become part of the spacetime structure, simplifying the mapping of the Dirac relativistic quantum representation into the eight dimensional, subalgebra of the GA spacetime representation. This allows a GA functional description of a photon. [1], which in turn allows a four-dimensional composite electron.

The authors previous paper [1], proposed a model of an electron formulated as the composition of two photons using the AG rotor structures for QM formulated by Doran et.al. [2]. (Fig. 1).

This presentation offers an electron model in that context having similarities of the atomic physical model, but relies on the gradients in the index of refraction produced by the nonlinear effects of vacuum polarization as the binding mechanism.

Sketches

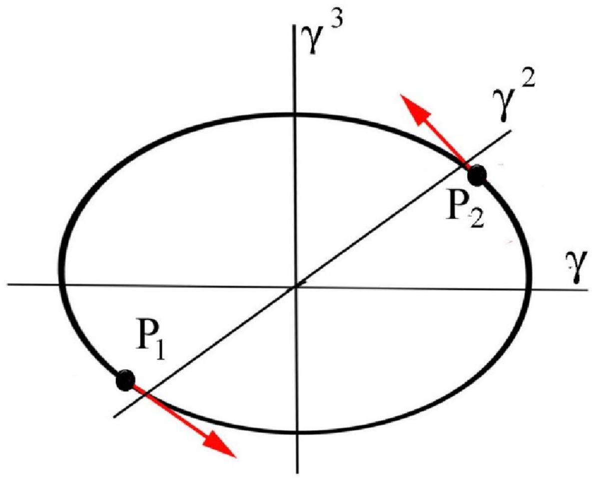

Fig.1 General configuration showing the orbiting of photons in a GA cooridnate syatem

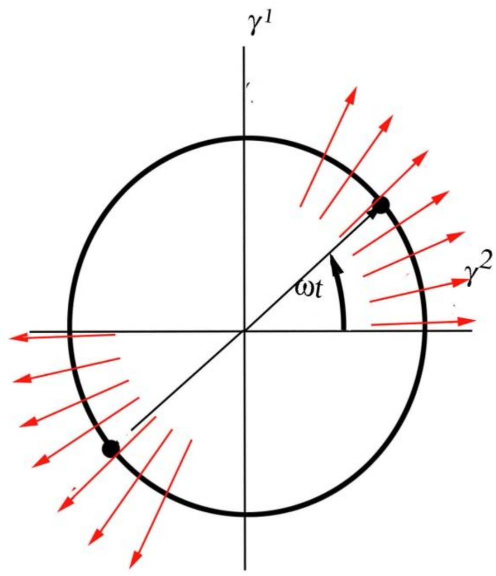

Fig.2. Radially polarized photons bound by the self-generated vacuum polarization gradient, maintaining a radially polarized circular electric vector probability.

The electric vector does not constitute a charge, but a polarization of the photon flow probability. The Lorentz transformation properties that constitute the magnetic vectors are determined by its internal rotations and are opposite for photons of opposite chirality.

Structure of the article

Primary Physical Mechanisms

The interaction of the two orbiting photons (Fig.1) as described in the earlier paper is not the result of an electric force, but by the motion under the influence of the mutual gradient in the index of refraction. This gradient is generated by the self-induced vacuum polarization of the two opposite going photons.

The vacuum polarization effect between two interaction photons is a well-researched process both from theoretical and experimental aspects. The first

development by Sauter, Serber, Euler and others,[3],[4],[5], and later by more sophisticated methods of QFT by Schwinger and others[8],[9].
The study of the vacuum polarization on the index of refraction is quite extensive in the lower levels of E when birefringence on photons in static fields effects are predominant, [10],[11],[12],[13],[14],[15],[16],[17],[18],[19],[20],[21]. Others have studied and proposed experiments investigated the effects of photon-photon scattering in the higher energy levels, ,[22],[23], and there have been several proposals for studies on the effects on the index of refraction by intense laser beams,[24],[25],[26],[27],[28],[29].

At the higher E levels, that are more appropriate to this work, the processes of Delbruck scattering and pair production dominates. These processes, originally proposed by Max Delbruck and first observed, by Robert Wilson [30], have been the subject of intensive research in both theoretical, and experimental since the 1950's. [31],[32], [33],[34],[35],[36], [37]. Appropriate to this paper, but not at the same energy levels is the research done by J. Kim et.al. [38], on light bending in a Coulombic field.

Vacuum polarization and the Index of Refraction

The most important aspect has been the derivation by Schwinger of the leading nonlinear corrections to the vacuum polarization that allows calculations of the local index of refraction below the critical electron–positron limit [3].

$$E_{CR} = \frac{m_e^2 c^3}{Q\hbar} = \frac{c\hbar}{Q\lambda\!\!\!^{-}_e{}^2} \qquad (1,1)$$

At the low-energy end with non-parallel fields generally defined by the Heisenberg-Euler Lagrangian are the studies of birefringence changes in the index of refraction induced at low levels $\left(E \ll E_{cr}\right)$ .These have been conducted by a large number of researchers [10-21], and the results are generically similar to:

$$\eta^{\parallel,\perp} = 1 + \frac{\alpha(11 \mp 3)}{45\pi}\frac{E_2^2}{E_{cr}^2} \tag{1,2}$$

The $\parallel,\perp$ suffix indicates parallel and perpendicular field polarizations.

For two photons moving around a common center of momentum each experiences the electromagnetic field of the other. The relation for that interaction at $E << E_{CR}$ from Kim et.al, "Light bending in radiation background" [38], and Light bending in a Coulombic field the index of refraction can be expressed as:

$$\eta^{-1} = \frac{c}{c_0} = 1 - \frac{(14\perp,8\parallel)\alpha^2\hbar^3}{45m^4c^5}\left(u \times E_2\right)^2 \tag{1,3}$$

At the higher end of the energy levels above the Kim et.al, work closer to the Schwinger limit $\left(E \sim E_{cr}\right)$, the index of refraction is better understood and by the processes related to Delbruck scattering, and pair production.

The reflection coefficient expressed in the relative index of refraction and the high end scattering experiments, lead to the conclusion that the index of refraction has an infinity at the Schwinger Limit. With multiple loops and higher order corrections the index of refraction at the higher fields as developed by Dietrich et.al. ([12] Fig,1) is: $E \to E_{cr}$ index of refraction η^{-1} is:

$$\sqrt{\eta^{-2}} \to \left(1 - Q\frac{B^2}{2B_{cr}^2}\sin^2\theta\right) \tag{1,4}$$

At very high levels of the fields the Q factor $\to 1/2$, and if E represent the maximum of the E & B fields of photons near the Schwinger limit then for two opposite colliding photons with fields of E_1 and E_2 the maximum local index of refraction is:

$$\eta^{-1} \to \left(1 - \frac{E_1 \bullet E_2}{2E_{cr}^2}\right) \tag{1,5}$$

Index of Refraction for Photons containment

The index of refraction to maintain photons in a circular path can be determined from classical physics by variation methods applied to Fermat's principle. It is straight forward and well done by J. Evans, et.al. [39], and for stable orbits Fermat's principle requites the index of refraction to be proportional to1/r, thus in terms of the Compton radius for an electron, This can be written as:

$$\frac{c}{c_0} = \frac{kr}{\lambda_e} \rightarrow \frac{\Delta c}{c_0} = 1 - \frac{kr}{\lambda_e} \qquad (1,6)$$

k is the index of refraction constant and λ_e is the Compton radius of the electron

Putting this into Eq.(1,5), the relations between c, r and E is:

$$\eta^{-1} = \left(\frac{c}{c_0}\right) = \left(\frac{kr}{\lambda_e}\right) = \left(1 - \frac{E^2}{E_{cr}^2}\right) \qquad (1,7)$$

The value of k can be determined from Eq.(1,6), and by appealing to the Electron spin of the two orbiting photons that make up the electron.

$$(m_1 + m_2)cr = \frac{\hbar}{2} \rightarrow cr = \frac{\hbar}{2m_e} \rightarrow \frac{\hbar c_0}{2m_e c_0} = \frac{\lambda_e c_0}{2} \qquad (1,8)$$

(1,9)

The momentum $p = mc$,as well as the angular momentum are a function of c thus the index of refraction η^{-1} is

$$\frac{c}{c_0} = \frac{\lambda_e}{2r} \qquad (1,10)$$

The ratio of c/c_0 from the requirement for stable photon orbits Eq.(1,6), and the electron spin and Eq.(1,10), must be equal at the photon primary orbit.

$$\frac{c}{c_0} = \frac{\lambda_e}{2r} \quad , \quad \frac{r}{2\lambda_e} = \frac{c}{c_0} \qquad (1,11)$$

Eq.(1,6), and Eq.(1,10), have a solution at $k == 1/2$, and the primary orbit for the photons r_p is the Compton radius for the electron.

$$r_p = \lambda\!\!\!^{-}_e \tag{1,12}$$

From Eq.(1,6), At the primary orbit the value of Δc is :

$$\frac{\Delta c}{c_0} = \frac{1}{2} \tag{1,13}$$

The two photons orbit at the Compton radius of the electron, exactly the expected radius of the electron, (Fig. 3.).

Photon Velocity vs Orbital Radius

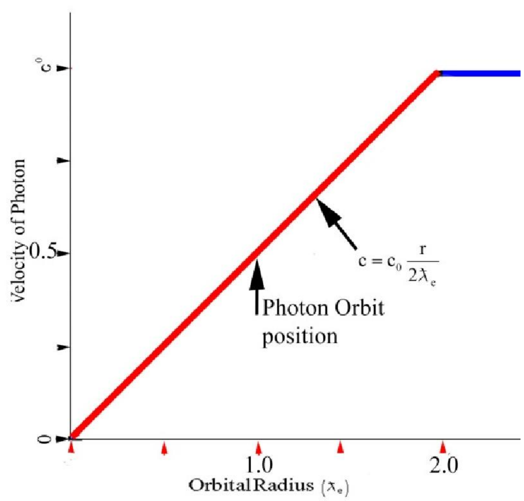

Fig. 3. This is the velocity of light experienced by the photons in orbit around the center of momentum as the result of vacuum polarization, and the most probable position of the orbit.

Connection to Schwinger Vacuum Polarization

If the centrifugal force on the orbiting photons is set equal to an electric force, the electric field in the electric field intensity in the electron can be evaluated from the centrifugal force and the orbital radius. Eq.(1,12), the radius is the Compton radius of the electron, thus.

$$f_c = \frac{m_1 c^2}{r} \rightarrow \frac{mc^2}{\frac{\hbar}{m_e c_0}} = \Rightarrow \frac{m_2 c^3}{\hbar} \tag{1,14}$$

If this is then set to be the electric field force:

$$QE \to \frac{m_c^2 c^3}{\hbar} \to E = \frac{m_c^2 c^3}{Q\hbar} = E_{cr} \qquad (1,15)$$

I The value of E ,or the electric density to hold the photon in place if it were charged is the **same as the value of the Schwinger critical Electric intensity for pair production** Eq.(1,1). Putting this into Eq. (1,5), gives the value of the index of refraction which is the index of refraction at the photon orbit:

$$\eta^{-1} \to \left(1 - \frac{E_{cr}^2}{2E_{cr}^2}\right) = \frac{1}{2} \qquad (1,16)$$

For clarification of the relation between c and η^{-1}

$$\left(1 - \eta^{-1}\right) = \frac{c_0 - c}{c_0} = \frac{\Delta c}{c_0} = \frac{1}{2} \qquad (1,17)$$

This calculation should not be taken too seriously since the change in c in the frame of the photon is actually due to the apparent increase in density of the background Feynman photons due to the photon orbital rotation.

Photon Radial Alignment: Thomas Precession

Thomas Precession is a well understood phenomenon totally within the mechanics of Lorentz dynamics.
As a particle rotates around a center axis there is a frame rotation such that when it arrives back at a defined point its helictical phase orientation will also have been rotated. For a photon in a circular index of refraction this will mean that for every cycle of rotation about the axis its helictical rotation will be reduced by that number of cycles. At half the Compton radius of the photon, this reduces the helictical frequency to zero leaving the photon with a constant radial electric vector. See **Appendix I** for details.)

Conclusion

A model for an electron has been presented that physically demonstrates mass, , spin, & polarization within the concepts of currently known physics. Nothing has been postulated that isn't well understood in terms of current physics. Neither of the physical regimes of QM or Classical physics have been stretched, compromised, or extended beyond that which has been experimentally confirmed.

It gives a physical insight to the mechanical process, and since there are no singularities associated with the model. This structure allows QFT a path around the infamous renormalization without having to cancel infinite values with infinite values.

The Electron-Electron interactions as the result of the external Feynman Photon-photon interactions are included in "A Quantum Theory Conjecture on the Origin of Gravitational and Electric Particle Interaction"

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Appendix I

Thomas Precession

As a pair of photons rotates around the center of a circle due to a variable index of refraction, the Thomas precession reduces the helictical rotation frequency of the photon. The photons frequency is reduced by exactly the axial frequency of the rotation. As the gradient in the index of refraction is increased the sum of the frequencies must remain constant.

As the circumference is reduced to the wavelength the helictical frequency is stopped. The rotation frequency is then equal to the original free particle frequency of the photon and the photon electromagnetic vectors are polarized along the orbital radius.

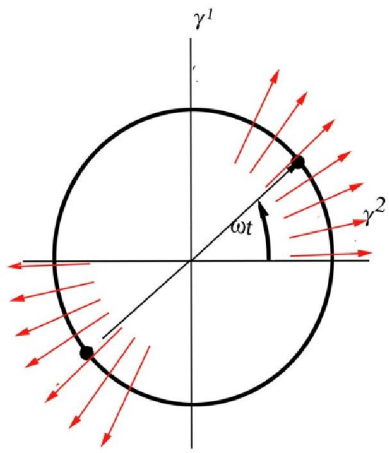

Fig.2 Two radially polarized photons bound by the self-generated vacuum polarization gradient in the index of refraction. The radially polarized circular directed electric vectors constitute the effect of a local charge.

This is easily shown from Lorentz geometric principles, the Thomas reduction to the frequency of an orbiting photon is:

$$\omega_T = \frac{1}{c^2}\left(\frac{\gamma^2}{\gamma+1}\right) a \times v \qquad (1,18)$$

a is the circular acceleration dr/dt in the moving frame thus:

$$\Delta t' = \gamma \, \Delta t \qquad (1,19)$$

and for a photon moving in a variable index of refraction the precession is:

$$\vec{\omega}_T = \frac{1}{c^2}\left(\frac{\gamma^2}{\gamma+1}\right) a \times v \rightarrow \frac{1}{c^2}\left(\frac{\gamma^2}{\gamma+1}\right)\frac{dv}{\gamma dt} \times v = \frac{1}{c^2}\left(\frac{\gamma^2}{\gamma+1}\right)\frac{dv}{dt} \times v$$

For the photon the circular acceleration is:

$$\frac{dv}{dt} = \frac{c^2}{r} \qquad (1,20)$$

And for the photon orbiting at the Compton radius:

$$r = \frac{c}{\omega_P} \qquad (1,21)$$

Thus as the radius is reduced to the Compton radius the Thomas precession frequency reduces the helictical frequency to zero, whereas the axial frequency in the orbit plane $\vec{\omega}_R$ becomes equal to the free photon frequency.

$$\vec{\omega}_T = \omega_P \uparrow \qquad (1,22)$$

The Thomas precession thus establishes a radial polarization if the electric vector along the radius vector to the center of momentum.